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B.M.S COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU – 560004

SEMESTER END EXAMINATION – SEPT/OCT-2023

M.Sc in Mathematics – 2nd Semester

COMPLEX ANALYSIS

Course Code: MM202T **QP Code: 12002**

Duration: 3 Hours **Max. Marks: 70**

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. (a)State and prove Cauchy's theorem for a disk.

   (b) Evaluate (i) $\int_C \frac{e^z}{z(z-1)(z-2)} dz$; $C: |z| = 1.5$ (ii) $\int_C \bar{z}^2 \, dz$; $C: |z-1| = 1$

   (c) State and prove the Cauchy integral formula and use it to evaluate

$$\int_C \frac{\cos 2\pi z}{(2z-1)(z-3)} dz; \quad C: |z| = 1$$

(5+5+4)

2. (a) Let $f(z)$ be analytic in a region D of the complex plane and $f(z) \neq 0$ in D If a is a zero of $f(z)$ in D then there exist a positive integer $'n'$ and an analytic function $g(z)$ on D such that $f(z) = (z-a)^n g(z)$ when $g(z) \neq 0$.

   (b) State and prove Liuoville's theorem. Deduce the fundamental theorem of algebra.

(7+7)

3. (a) Find the radius of convergence of (i) $\sum_{n=1}^{\infty} \frac{n\sqrt{2}+i}{1+2in} z^n$ (ii) $\sum_{n=1}^{\infty} (3+4i) z^n$.

   (b) State and prove Taylor's theorem.

   (c) Let $f(z) = \sum_{n=1}^{\infty} a_n (z-a)^n$ in $\{|z-a| < R\}$ where R is radius convergence of the power series. Then prove that the Taylor's expansion of $f(z)$ in the neighbourhood of a point a is exactly the given power series. **(4+5+5)**

4. (a) Find the Laurent's expansion of $f(z) = 1/z(z^2 - 3z + 2)$ for the regions

$(i)\ \ 0 < |z| < 1, (ii) 1 < |z| < 2.$

(b) Let $f(z)$ be analytic function having as isolated singularity at $z = a$. If $|f(z)|$ is bounded in a neighbourhood $\{0 < |z - a| < r\}$ then prove that $f(z)$ has a removable singularity at $z = a$.

(c) State and prove open mapping theorem. **(5+5+4)**

5. Evaluate any three of the following (i) $\int_0^{\infty} \frac{\sin x}{x}\ dx = \frac{\pi}{2}$ (ii) $\int_0^{\infty} \frac{dx}{(x^2+1)^2}$

(iii) $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x(x^2-2x+2)} dx$ (iv) $\int_0^{\infty} e^{-x^2} \cos(2mx)\, dx = \sqrt{\frac{\pi}{2}}\ e^{-m^2}$.

(14)

6. (a) Let γ is a closed curve and $f(z)$ be meromorphic function defined inside and on the curve γ and $f(z)$ does not contain any zero's or pole's on γ. Let N be the number of zero's inside γ and let P be the number of poles of $f(z)$ inside γ .

Then $N - P = \frac{1}{2\pi i} \int_{\gamma} \frac{f\prime(z)}{f(z)} dz.$

(b) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2.$

(c) State and prove Maximum modulus theorem.

(5+4+5)

7. (a) State and prove Riemann mapping theorem.

(b) Let $f(z)$ be an entire function and $f(z) \neq 0$. Let $k \geq 0$ be order of zeros of $f(z)$ at $z = 0$. Let remaining of $f(z)$ be at z_1, z_2 …. where each z_n is repeated as often as it multiplicity. Then prove that $f(z) = e^{g(z)} z^k \prod_n EM_n\left(\frac{z}{z_n}\right).$

(7+7)

8. (a) State and prove Mean value theorem

(b) State and prove Poisson Integral formula.

(7+7)

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